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Investigation of Neutrino Mixing Angles with a Focus on Scaling and Hybrid Texture Models.

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Abstract

In this work, we explore the potential origins of various neutrino mixing angles by examining two distinct types of neutrino mass matrices. Specifically, we analyze the type I seesaw and type II seesaw mass matrices to understand their roles in shaping neutrino mixing angles and mass hierarchy. We treat the type I seesaw as a hybrid texture neutrino model and the type II seesaw as a scaling mass matrix. Our aim is to provide a concise overview of neutrino mass models, incorporating the latest insights into neutrino dynamics.ata.

Keywords: *neutrino, seesaw, scaling, mixing*

I. Introduction

Origin of tiny neutrino masses and their large mixing **[1-5]** is one of the major observed phenomena that the Standard model of particle physics would not explain. Various Neutrino oscillation experiments across the world, namely T2K [6] ,Double ChooZ [7] , Daya-Bay [8,] and RENO [9,] have made the earlier predictions for neutrino parameters precise precisely and also a predicted non-zero value of the reactor mixing θ_{13}. The latest global fit value for 3σ range of neutrino oscillation parameters are given in **[10]** and **[11].** The neutrino oscillation experiments measure only two mass squared differences and therefore the lightest neutrino mass, which remains a free parameter, can be constrained from the upper bound on the sum of absolute neutrino masses from cosmology, $\sum m_i <$ 0.12eV **[12].** In addition to the neutrino mass hierarchy problem, recent neutrino experiments have also not found anything about the nature of the neutrino mass. Therefore, these questions will lead to some different neutrino models to explain those undetermined neutrino parameters. People expected that the extreme smallness of the neutrino masses is most likely related to the seesaw mechanism. Some good works related to seesaw mechanism can be found in **[13, 14]**. All these mechanisms include extra heavy fermionic or scalar fields into the SM.

In this work, we consider a very specific neutrino mass matrix structure proposed a few years ago by the authors of [15-16]. The structure of the neutrino mass matrix is based on the idea of a strong scaling Ansatz, where certain ratios of the elements of the neutrino mass matrix are equal. We consider the scaling mass matrix as the origin of the type I seesaw mechanism in our neutrino model. We also consider some hybrid texture neutrino models [17-20] as the type II seesaw matrix. Here, we combine the scaling neutrino mass matrix and hybrid texture neutrino matrices to generate different neutrino parameters and attempt to correlate them. This paper is organized as follows: In the next section, II, we discuss different possible hybrid texture neutrino mass models and the scaling mass matrix. We discuss the numerical methods in section III. In section IV, we present our results and conclusions.

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II. Scaling mass matrix and hybrid texture neutrino mass matrix

We write the neutrino mass matrix and leptonic mixing matrix

$$M_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

In terms of matrix element. Type A scaling mass matrix is given by authors

$$M_\nu = \begin{pmatrix} A & B & \frac{B}{S} \\ B & D & \frac{D}{S} \\ \frac{B}{S} & \frac{D}{S} & \frac{D}{S^2} \end{pmatrix}$$

There are three types of scaling neutrino mass matrix **[15]** . Only case A is allowed by current data and in this work we consider the model based on case A only which is given above. If neutrinos are Majorana fermions, as predicted by the conventional seesaw mechanisms, then the 3×3 neutrino mass matrix is complex symmetric and hence has six independent complex elements. In literature review Several works related to one-zero and two-zero textures can be found in **[17]** and **[18]** respectively. Another possibility is the so-called hybrid texture defined as combination of one-zero texture and two equal non-zero elements **[17–20].** In this work we consider hybrid texture neutrino mass matrix as the origin of type I seesaw. There are six categories of hybrid texture matrix which make it 39 matrices. We choose only 6 out of 39 hybrid texture matrices in our work which is closely agreed with experimental values. Following are the structure of hybrid texture neutrino matrix we have used in our work.

$$\text{A1:} \begin{pmatrix} 0 & \times & \times \\ \times & \nabla & \nabla \\ \times & \nabla & \times \end{pmatrix}, \text{B1:} \begin{pmatrix} \nabla & 0 & \nabla \\ 0 & \times & \times \\ \nabla & \times & \times \end{pmatrix}, \text{C1:} \begin{pmatrix} \nabla & \nabla & 0 \\ \nabla & \times & \times \\ 0 & \times & \times \end{pmatrix}, \text{D1:} \begin{pmatrix} \nabla & \nabla & \times \\ \nabla & \times & 0 \\ \times & 0 & \times \end{pmatrix}$$

$$\text{E1:} \begin{pmatrix} \nabla & \nabla & \times \\ \nabla & 0 & \times \\ \times & \times & \times \end{pmatrix}, \text{F1:} \begin{pmatrix} \nabla & \nabla & \times \\ \nabla & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

III. Numerical Analysis

The neutrino mass matrix can be obtained by two mass square differences and mixing angles provided by data. The leptonic
mixing matrix or the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix is given by

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$$U_{PMNS} = {U_l}^{+} U_\nu$$

U_ν, U_l represent neutrino mass and lepton mixing matrix. if we assume charged lepton matrix as a diagonal then the light neutrino mass matrix can be written as

$$U_{PMNS} = {U_l}^{+}$$

Considering the type II seesaw term as the necessary correction to scaling mixing, we write the neutrino mass matrix as

$$M_\nu = M_I + M_{II} = m_D {M_{RR}}^{-1} {m_D}^T \ + M_{II}$$

Where M_I, M_{II} are the scaling and hybrid texture mass matrices. We can write neutrino mass matrix as

$$U_{PMNS} {m_\nu}^{diag} {U_{PMNS}}^T = scaling\ mass\ matrix\ + Hybrid\ texture\ matrix$$

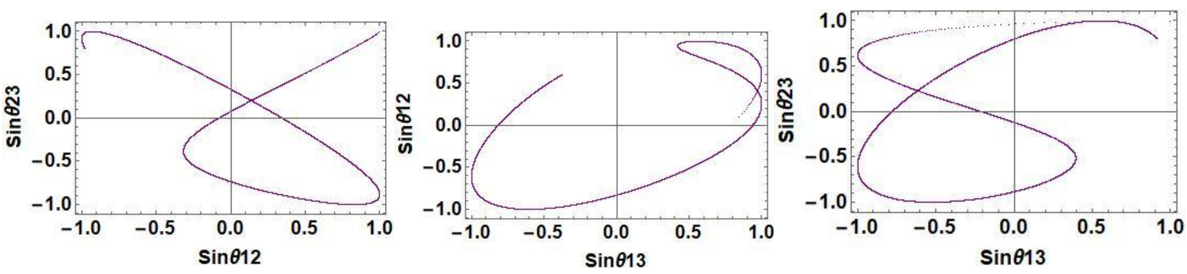
For normal mass hierarchy, the diagonal mass matrix of the light neutrinos can be written as

$${m_\nu}^{diag} = diag(m_1, \sqrt{{m_1}^2 + {\Delta m_{21}}^2}, \sqrt{{m_1}^2 + {\Delta m_{31}}^2})$$

whereas for inverted mass hierarchy it can be written as

$${m_\nu}^{diag} = diag(\sqrt{{m_3}^2 + {\Delta m_{23}}^2 - {\Delta m_{21}}^2}, \sqrt{{m_3}^2 + {\Delta m_{23}}^2}, m_3)$$

In this work, we have not considered the Majorana CP phases; we deal only with the Dirac CP phase of the neutrino mass matrix. We can take m1 as the lightest neutrino mass in the case of NH after substituting the values of mass squared differences [10-11], and in the IH case, we take m3 as the light neutrino mass eigenvalue. We can construct the neutrino mass matrix as a combination of a scaling neutrino mass and a hybrid texture neutrino mass. In this work, we choose some fixed values of the lightest neutrino mass within the cosmological limit given by [12]. In the case of a scaling mass matrix, we have solved for the values of A, B, S, D by comparing with the neutrino mass matrix elements. Now, by comparing each element of this newly constructed mass matrix with the original neutrino mass matrix, we can plot some graphs between different neutrino parameters, considering different hybrid textures models with a scaling matrix. We have shown a few plots of mixing angles from our calculations.



Hybrid texture model	NH	IH
A1	√	√
B1	√	×

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C1	×	×
D1	×	×
E1	×	×
F1	√	×

IV. Results and Conclusions

In this work, we have studied six hybrid texture neutrino mass matrices allowed by the latest experimental data. We also explore a different type of neutrino matrix known as the scaling strong matrix. In the hybrid texture neutrino mass matrix, there exists one zero and two equal non-zero elements. There are three possibilities for the scaling mass matrix; however, in our work, we consider only one that consistently yields an inverted hierarchy (IH) structure. We combine the hybrid texture and scaling mass matrices to construct a neutrino mass matrix that accommodates both normal and inverted hierarchy structures. Since the scaling mass matrix always produces a definitive inverted structure model, we attempt to introduce corrections to the scaling matrix using texture zeros in order to derive a complete neutrino mass matrix. Neutrino mass matrices can originate from different types of seesaw mechanisms. In this study, we consider type I and type II seesaw mechanisms as the origins of the neutrino mass matrix. We equate the type I seesaw with the scaling matrix and the type II seesaw with the hybrid texture. We investigate the variation of the lightest neutrino mass with neutrino mixing angles in both the normal hierarchy (NH) and inverted hierarchy (IH) cases. We have only obtained correct results for the three hybrid texture cases summarized in the table above. The check mark in the table indicates correct neutrino oscillation angles observed in the hybrid texture model, while the cross symbol denotes oscillation parameters that fall outside the acceptable range. From these results, we can constrain the texture models to align with experimental values. In this work, we focus solely on the Dirac CP phase. In the future, we aim to study results incorporating all three phases—Dirac and Majorana—which will be interesting for exploring neutrinoless double beta decay and leptogenesis with these new neutrino models.

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References

1. S. Fukuda, et al. “Constraints on neutrino oscillations using 1258 days of super-kamokande solar neutrino data”. *Phys. Rev. Lett. 86*, 5656-5660 (2001).
2. Q.R. Ahmad, et al. “Direct evidence for neutrino flavor transformation from neutral current interactions in SNO,” *Phys. Rev. Lett. 89*, 011301 (2002).
3. Ahmed, Q. R. Ahmad, et al. “Measurement f day and night neutrino energy spectra at SNO,” *Phys. Rev. Lett. 89*, 011302 (2002).
4. J.N. Bahcall, et al. “Solar modes and solar neutrino oscillations,” *New J. Phys. 6*, 63 (2004).

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5. K. Nakamura et al. " Review of particle physics," *Nucl. and part. Phys 37*, 075021 (2010).
6. K. Abe, et al. "Indication of electron neutrino appearance from an accelerator produced off-axis muon neutrino beam", *Phys. Rev. Lett. 107*, 041801 (2011).
7. Y. Abe, et al. " Indication for the disappearance of reactor electron antineutrinos in the double chooz experiment," *Phys. Rev. Lett. 108*, 13180 (2012).
8. F.P. An, et al. "Observation of electron antineutrino disappearance in the daya bay experiment," *Phys. Rev. Lett.* 108, 171803 (2012) .
9. J.K. Ahn, et al. "Observation of reactor electron antineutrino disappearance in the RENO Experiment," *Phys. Rev. Lett*. 108, 191802 (2012) .
10. P.F. de Salas, P.F., et al.. "Status of neutrino oscillations 2018," *Phys. Rev. D98*, 030001(2018).
11. I. Esteban, et al., " Global analysis of three flavor neutrino oscialltion," *JHEP* 01 106 (2019).
12. N. Aghanim, et al. "*Planck 2018 results. VI.*" HEP 01, 106 (2019).
13. , R.N. Mohapatra, et al." Neutrino mass and spontaneous parity violation ," *Phys. Rev. Lett 44*, 912(1980).
14. D.Borah, et al. " Derivations from tribimaximal neutrino mixing using type II seesaw," *Nucl. Phys. B876*, 575 (2013).
15. R. N. Mohapatra and W. Rodejohann, Phys. Lett. B 644, 59 (2007).
16. R. Kalita, D. Borah and M.K. Das , *Nucl. Phys. B894*, 307 (2015).
17. S. Kaneko, H. Sawanaka and M. Tanimoto, JHEP 0508, 073 (2005);
18. S. Dev, S. Verma and S. Gupta, Phys. Lett. B687, 53 (2010);
19. S. Goswami, S. Khan and A. Watanabe, Phys. Lett.B693, 249 (2010).
20. R. Kalita and D. Borah, Int. J. Mod. Phys. A31, 1650008 (2016).